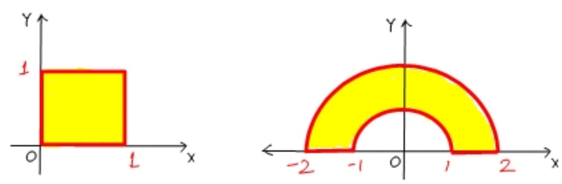
Computational Topology (Spring 2018) — Homework 1

- You **must email your submission** as a **PDF file** to bkrishna@math.wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
- Your file name should identify you. For instance, if you are Leornado Di Cartman, you should name your submission LeornadoDiCartman_Hw1.pdf. **Please start your name in this format. If you want to add more bits to the title, e.g., Math574, you could name it LeornadoDiCartman_Math574_Hw1.pdf, for instance. Also, please avoid white spaces in the file name :-).**
- **This homework is due before midnight on Thursday, January 25**.

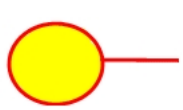
1. (20) Show that the following two sets are homeomorphic by explicitly specifying a homeomorphism, i.e., a continuous function f from one set to the other, and its inverse.



2. (25) Let us define a *neighborhood* of a point as any *open* set that contains the point. Consider a set A that is a subset of $\mathbb{R}^2$. We define a point $\mathbf{x} \in \mathbb{R}^2$ is **near** the set A if every neighborhood of $\mathbf{x}$ contains a point of A, (i.e., intersects A). We denote this definition by $\mathbf{x} \leftarrow A$.

   Let A, B be sets in $\mathbb{R}^2$, and $\mathbf{x} \in \mathbb{R}^2$. Prove that if $\mathbf{x} \leftarrow A$ or $\mathbf{x} \leftarrow B$, then $\mathbf{x} \leftarrow A \cup B$. Also prove the converse, i.e., that if $\mathbf{x} \leftarrow A \cup B$, then either $\mathbf{x} \leftarrow A$ or $\mathbf{x} \leftarrow B$, or both.

3. (20) State if each of the following objects is a manifold, a manifold with boundary, or neither. If it is one of the first two cases, specify the dimension of the manifold.

(a)  (b)