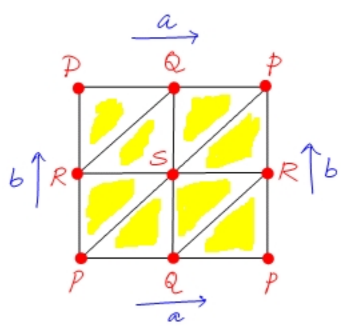
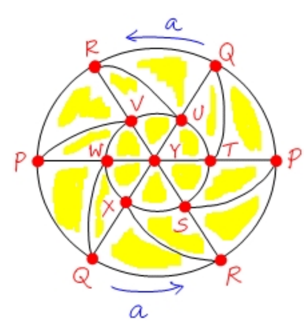
Computational Topology (Spring 2018) — Homework 2

- You **must email your submission** as a **PDF file** to bkrishna@math.wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
- Your file name should identify you. For instance, if you are Hean Sannity, you should name your submission HeanSannity_Hw2.pdf. **Please start your name in this format. If you want to add more bits to the title, e.g., Math574, you could name it HeanSannity_Math574_Hw2.pdf, for instance. Also, please avoid white spaces in the file name :-).**
- **This homework is due before midnight on Tuesday, February 6**.

1. (35) List all the ways in which the sides of a rectangle can be identified in pairs. In each case, indicate which of the surfaces introduced in class (in Lectures 4 and 5) if any, does the resulting object represent (we saw the 2-sphere ($\mathbb{S}^2$), torus ($\mathbb{T}^2$), Möbius strip, projective plane ($\mathbb{R}P^2$), and the Klein Bottle ($\mathbb{K}^2$).

2. (20) The following are *potential* triangulations of the torus $\mathbb{T}^2$ and the real projective plane $\mathbb{R}P^2$, respectively. Decide if they are indeed correct triangulations of the two spaces. Justify your answers.

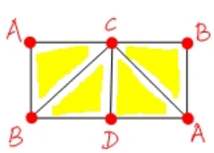




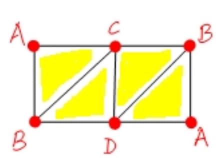
3. (30) A *flag* in a simplicial complex K in $\mathbb{R}^d$ is a nested sequence of proper faces $\sigma_0 \prec \sigma_1 \prec \cdots \prec \sigma_k$. The collection of flags in K forms an abstract simplicial complex A, called the *order complex* of K. Prove that the order complex A of K has a geometric realization in $\mathbb{R}^d$.

4. (30) Describe the space represented by each of the following three triangulations. Also calculate the Euler characteristic χ in each case, and compare it to the χ values of standard 2-manifolds we discussed in class ($\mathbb{S}^2, \mathbb{T}^2$, etc.).

(a)



(b)



(c)