Module 1: Whole Numbers

In this module, we will learn about place value, how to write a number using digits or words, how to add, subtract, multiply, and divide whole numbers, how to find factors and multiples, and how to round whole numbers.

I. PLACE VALUE AND WRITING WHOLE NUMBERS

First of all, let’s determine what a whole number is. We can also call whole numbers the counting numbers, so they are:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, ...

Note that numbers like 4.2 or $11/3$ are NOT whole numbers.

Numbers can be written using digits. A digit is a single character in our numbering system (the first ten whole numbers are digits). We combine digits to form other numbers such as 23. Note that putting the digits 2 and 3 together in a different order, 32, creates a different number. This is because of place value. When we write 23, this means we have two “tens” and three “ones.” When we write 32, this means we have three “tens” and two “ones.” Below, we look at what 2,394,865 would look like in expanded form.

\[ 2,394,865 = (2 \times 1,000,000) + (3 \times 100,000) + (9 \times 10,000) + (4 \times 1,000) + (8 \times 100) + (6 \times 10) + (5 \times 1) \]

This means that because the 2 is in the “millions” place, it represents $2 \times 1,000,000$ or 2,000,000. Similarly, the 3 represents 300,000 as it is in the “hundred-thousands” place, the 9 represents 90,000 because it is in the “ten-thousands” place, and the 4 is in the “thousands” place and represents 4,000. Additionally, the 8 is in the “hundreds” place and represents 800, the 6 is in the “tens” place, so it represents 60, and the 5 is in the “ones” place so it represents five.

Now that we know what each numeral represents, we can write the number in words. Reading the number from left to right, we can write 2,394,865 as:

Two million, three hundred ninety-four thousand, eight hundred sixty-five.

Now, you can practice writing numbers in words. Answers are listed before the exercises.

You Try 1: Write each number in expanded form and in words.

a. 25,864  
b. 389,502  
c. 7,100,623

II. ADDING WHOLE NUMBERS

When adding whole numbers, be sure to line up digits that are in the same place value. For instance, 45 + 31 would look like:

\[
\begin{array}{c}
45 \\
+ 31 \\
\hline
76
\end{array}
\]
Now, we can add the digits in the ones column.

\[
\begin{array}{c}
45 \\
+ 31 \\
\hline
6
\end{array}
\]

Then, add the digits in the tens column.

\[
\begin{array}{c}
45 \\
+ 31 \\
\hline
76
\end{array}
\]

So, \(45 + 31 = 76\). Note that we may also say that the sum of 45 and 31 is 76.

What if we ended up with more than 9 when we add? We only have room for one digit, so what happens to the other digit? We must carry it to the next column. If we end with 13 “ones,” then we must write down the three and carry the one “ten” to the next column. Adding \(46 + 27\), we would first line up the digits:

\[
\begin{array}{c}
46 \\
+ 27 \\
\hline
73
\end{array}
\]

Now, we can add the digits in the ones column, writing down the three and carrying the one.

\[
\begin{array}{c}
146 \\
+ 27 \\
\hline
3
\end{array}
\]

Then, add the digits in the tens column, making sure to include the one that was carried.

\[
\begin{array}{c}
46 \\
+ 27 \\
\hline
73
\end{array}
\]

The process is similar for larger numbers. For example, \(628 + 75\) would look like:

\[
\begin{array}{c}
628 \\
+ 74 \\
\hline
628
\end{array}
\Rightarrow\begin{array}{c}
628 \\
+ 74 \\
\hline
742
\end{array}
\Rightarrow\begin{array}{c}
1628 \\
+ 74 \\
\hline
7402
\end{array}
\Rightarrow\begin{array}{c}
1628 \\
+ 74 \\
\hline
1674
\end{array}
\Rightarrow\begin{array}{c}
1628 \\
+ 74 \\
\hline
1702
\end{array}
\]

We can also add more than two numbers together at a time using a similar process:

\[
\begin{array}{c}
835 \\
423 \\
+ 517 \\
\hline
835
\end{array}
\Rightarrow\begin{array}{c}
835 \\
423 \\
+ 517 \\
\hline
835
\end{array}
\Rightarrow\begin{array}{c}
835 \\
423 \\
+ 517 \\
\hline
835
\end{array}
\Rightarrow\begin{array}{c}
835 \\
423 \\
+ 517 \\
\hline
835
\end{array}
\Rightarrow\begin{array}{c}
835 \\
423 \\
+ 517 \\
\hline
1775
\end{array}
\]

We can also add more than two numbers together at a time using a similar process:
Practice adding whole numbers. Answers are listed before the exercises.

You Try 2: Add.

a. \[ 32 + 58 \]
   \[ 80 \]

b. \[ 437 + 364 \]
   \[ 801 \]

c. \[ 541 + 136 \]
   \[ 677 \]

III. SUBTRACTING WHOLE NUMBERS

When we subtract whole numbers, we will again need to line up the digits according to place value. The subtraction \( 87 - 52 \) becomes:

\[
\begin{array}{c}
87 \\
- 52 \\
\hline
35 \\
\end{array}
\]

Now we subtract the ones column; take 2 away from 7:

\[
\begin{array}{c}
87 \\
- 52 \\
\hline
35 \\
\end{array}
\]

And, subtracting the tens column, we get:

\[
\begin{array}{c}
87 \\
- 52 \\
\hline
35 \\
\end{array}
\]

So, \( 87 - 52 = 35 \), and we can say that the difference of 87 and 52 is 35.

The process is slightly more complicated when we have a larger digit in the bottom row. We will need to “borrow” from the column on the left in order to complete the subtraction. Let’s use expanded notation to help us understand the process. Let’s find the difference between 63 and 28. We can write this:

\[
63 - 28 \Rightarrow \left[ (6 \times 10) + (3 \times 1) \right] - \left[ (2 \times 10) + (8 \times 1) \right]
\]

We cannot take 8 “ones” away from 3, so we must borrow a “ten” as shown:

\[
\left[ (6 \times 10) + (3 \times 1) \right] - \left[ (2 \times 10) + (8 \times 1) \right] \Rightarrow \left[ (5 \times 10) + (13 \times 1) \right] - \left[ (2 \times 10) + (8 \times 1) \right]
\]

And, completing the subtraction we get:
\[
\frac{(5 \times 10) + (13 \times 1)}{(2 \times 10) + (8 \times 1)}
\]

More commonly you will see the short form:

\[
\begin{array}{cccc}
63 & 563 & 563 & 563 \\
-28 & \Rightarrow & -28 & \Rightarrow & -28 & \Rightarrow & -28 \\
\hline
5 & 35 & 35 & 35
\end{array}
\]

Now you can practice subtracting whole numbers. Answers are listed before the exercises.

You Try 3: Subtract. Show any necessary borrowing.

a. \[93 \quad \quad \quad 61\]
b. \[72 \quad \quad \quad 54\]
c. \[325 \quad \quad \quad 218\]

IV. MULTIPLYING WHOLE NUMBERS

We will start our section on multiplication with an explanation of what multiplication is. When we see \(3 \times 4\) or say three times four, this means we want to add three fours together:

\[3 \times 4 = 4 + 4 + 4 = 12\]

Similarly, \(3 \times 15\) is adding three 15s:

\[3 \times 15 = 15 + 15 + 15 = 45\]

We can also think about \(3 \times 15\) in expanded form. This would look like:

\[3 \times 15 = 3 \times [(1 \times 10) + (5 \times 1)] = (3 \times 10) + (15 \times 1) = (3 \times 10) + [(10 \times 1) + (5 \times 1)] = (4 \times 10) + (5 \times 1) = 45\]

Another way to write this is to line up the digits just as we did with addition and subtraction. We can write:

\[
15 \\
\times 3
\]

Now, multiply 3 times the ones digit. We get \(3 \times 5 = 15\), so we write down the 5 and carry the 1 to the tens place, similar to the step above where we grouped \(10 \times 1\) with \(3 \times 10\):

\[
15 \\
\times 3 \Rightarrow 15 \\
\times 3 \Rightarrow 5
\]

Finally, multiply the 3 by the tens digit and add the 1 that was carried. We get \(3 \times 1 + 1 = 4\), just as we got \(3 \times 10 + 10 \times 1 = 4 \times 10\) above:

\[
15 \\
\times 3 \Rightarrow 15 \\
\times 3 \Rightarrow 5
\]
Let's follow along with an example containing larger numbers. Start with multiplying each digit in the top number by the ones digit in the bottom number:

\[
\begin{array}{c}
382 \\
\times 125 \\
\downarrow
\end{array} 
\Rightarrow \begin{array}{c}
3^{182} \\
\times 125 \\
\downarrow
\end{array} \Rightarrow \begin{array}{c}
4^{382} \\
\times 125 \\
\downarrow
\end{array} \Rightarrow \begin{array}{c}
4^{382} \\
\times 125 \\
\downarrow
\end{array}
\]
\[\begin{array}{c}
0 \\
10 \\
1910
\end{array}
\]

Now we multiply each digit in the top number by the tens digit in the bottom. Place the numbers in the row below the 1910. Note that we are multiplying by 20 rather than 2, so we will use a 0 in the ones column to indicate this. The rest of the process is the same:

\[
\begin{array}{c}
382 \\
\times 125 \\
1910
\end{array} \Rightarrow \begin{array}{c}
3^{82} \\
\times 125 \\
1910
\end{array} \Rightarrow \begin{array}{c}
1^{382} \\
\times 125 \\
1910
\end{array} \Rightarrow \begin{array}{c}
1^{382} \\
\times 125 \\
1910
\end{array}
\]
\[\begin{array}{c}
40 \\
640 \\
7640
\end{array}
\]

Next, multiply by the 1, which represents 100. We will use 00 in the ones and tens columns to indicate this:

\[
\begin{array}{c}
382 \\
\times 125 \\
1910 \\
7640
\end{array} \Rightarrow \begin{array}{c}
3^{82} \\
\times 125 \\
1910 \\
7640
\end{array} \Rightarrow \begin{array}{c}
1^{382} \\
\times 125 \\
1910 \\
7640
\end{array} \Rightarrow \begin{array}{c}
1^{382} \\
\times 125 \\
1910 \\
7640
\end{array}
\]
\[\begin{array}{c}
200 \\
8200 \\
38200
\end{array}
\]

Finally, add the numbers to get the product of 382 and 125:

\[
\begin{array}{c}
382 \\
\times 125
\end{array}
\]
\[1910
\]
\[7640
\]
\[38200
\]
\[47750
\]

We see that \(382 \times 125 = 47750\).

Now practice what you have learned. Answers are listed before the exercises.

You Try 4: Multiply. Show all steps.

a. \(\frac{35}{8}\)  
b. \(\frac{47}{62}\)  
c. \(\frac{615}{27}\)
V. DIVIDING WHOLE NUMBERS

To understand division, we must understand the relationship between multiplication and division. For every set of two numbers that create a product such as \( 9 \times 6 = 54 \), we can rewrite the equation to show the relationship using division. Consider the family of equations below:

\[
\begin{align*}
9 \times 6 &= 54 \\
6 \times 9 &= 54 \\
54 \div 9 &= 6 \\
54 \div 6 &= 9
\end{align*}
\]

If any one of the equations is true, then every one of the equations is true.

When we are trying to divide larger numbers, such as \( 347 \div 4 \), we might want to write them as shown below so it is easier to show our work.

\[
4)347
\]

In order to divide, we will first ask how many times 4 divides into 3. Since 4 does not go into 3, we will now look at 34. We see that \( 4 \times 8 = 32 \), so 4 divides into 34 just over 8 times. We write the 8 above the “34” and the 32 below. We then subtract the 32 from the 34 and “drop down” the 7:

\[
\begin{align*}
8 & \quad \Rightarrow \quad 8 \\
4 \div 347 & \Rightarrow \quad -32 \downarrow \\
& \quad \Rightarrow \quad 8 \\
& \quad \Rightarrow \quad -32 \downarrow \\
& \quad \Rightarrow \quad 27
\end{align*}
\]

Next, determine how many times 4 divides into 27. We see that \( 4 \times 6 = 24 \), so 4 divides into 27 six times with some left over. We write:

\[
\begin{align*}
8 & \quad \Rightarrow \quad 8 \\
4 \div 347 & \Rightarrow \quad -32 \downarrow \\
& \quad \Rightarrow \quad 27
\end{align*}
\]

Because there are no more digits to “drop down,” we can no longer repeat the process. Thus, 86 is the quotient and 3 is a remainder.

Now, you can practice division. Answers are listed before the exercises.

You Try 5: Divide. State the quotient and remainder.

a. \( 48 \div 6 \)
b. \( 251 \div 8 \)
c. \( 5527 \div 24 \)
d. \( 1048 \div 5 \)
VI. FACTORS AND MULTIPLES

We now have the tools to write whole numbers in terms of their factors, which will be useful in subsequent modules. A factor is a whole number that divides another whole number evenly. For instance, the numbers 3 and 4 are factors of 12, because $3 \times 4 = 12$. The numbers 1, 2, 6, and 12 are also factors of 12 because $2 \times 6 = 12$ and $1 \times 12 = 12$. There are no other whole numbers that will divide 12 evenly, so the factors of 12 are 1, 2, 3, 4, 6, and 12.

What are the factors of 15? Thinking about what will multiply to 15, we recall that $3 \times 5 = 15$ and $1 \times 15 = 15$.

What are the factors of 32? There are several pairs of numbers that will multiply to 32, so let's list them systematically. Start with the smallest factor, 1, and then move on to check 2, then 3, etc. We recognize that $1 \times 32 = 32$, $2 \times 16 = 32$, 3 does not divide 32 evenly, $4 \times 8 = 32$, 5 does not divide 32 evenly, 6 does not divide 32 evenly, and neither does 7. We have already determined that 8 divides 32, so at this point we would just be repeating ourselves. Note that $5 \times 6 = 30$ and that every pair of whole numbers (that we have not already determined to be factors of 32), will multiply to something larger than 32. We could have ended our search for factors of 32 at 6.

Next we will discuss a couple of definitions. A prime number is a whole number greater than 1 that is divisible by only 1 and itself. We saw that the examples above were all divisible by at least one additional pair of numbers, so none of them were prime. Can you list a few prime numbers? We can start by considering 2, as it is the smallest whole number greater than 1. Since 2 is divisible by only 1 and itself, we say 2 is prime. And 3 is also prime. What about 4? Since 4 is divisible by 2, we know that it is not prime. Here is a list of some primes:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, ...$$

Is there anything you notice about the kinds of numbers on the list? Such as 2 is the only even prime? This must be true, because every even number larger than 2 is divisible by 1, itself, AND 2. Thus it would not be prime.

So what do we call a number that is not prime? Any whole number greater than 1 that is not prime is composite. According to what we determined above, 12, 15, and 32 are all composite. Here is a list of some of the composite numbers:

$$4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, ...$$

Note that any whole number greater than 1 is either prime or composite.

Now that we can find the factors of whole numbers and we know what the difference between prime and composite is, we are able to also determine the prime factorization of whole numbers. Let's go back to our example of 15. We noted that $3 \times 5 = 15$. Since both 3 and 5 are prime numbers, we can say $15 = 3 \cdot 5$ is the prime factorization of 15, or the way we would factor 15 into a product of all primes. [Please note that 1 will never be part of a prime factorization as it is NOT prime!]

What is the prime factorization of 12? We noted that $2 \times 6 = 12$ and $3 \times 4 = 12$, neither of which has all prime numbers. We can continue to factor the composite numbers to determine the prime factorization of 12. If we consider $2 \times 6 = 12$, we see that 2 is prime but 6 is composite. We can factor 6 to get $2 \times 3 = 6$, so we can write the prime factorization of 12 as $2 \times 2 \times 3$. Note that if we had considered $3 \times 4 = 12$, we would factor 4 to get $2 \times 2 = 4$, and we would end up with the same prime factorization. In fact, no matter how you determine a number's prime factorization, it will always be the same.
Note that 12 has two factors of 2. We can write this in shorter form as \(2 \times 2 = 2^2\), where the superscript 2 represents how many factors of two we want to multiply together. The superscript 2 is called the **exponent**.

For more practice, let’s determine the prime factorization of 32. Above we determined that \(4 \times 8 = 32\). Neither 4 nor 8 are prime, so let’s factor both of them. We would get \(2 \times 2 = 4\) and \(2 \times 4 = 8\), so \(32 = 2 \times 2 \times 2 \times 4\). We have already factored 4, so substituting in for 4 we have \(32 = 2 \times 2 \times 2 \times 2 = 2^5\). Again, the superscript 5 is the exponent, which means we have five factors of 2 multiplied together.

In the last part of this section, let’s talk about **multiples** of whole numbers. The multiples of a whole number will be found by multiplying that whole number by other whole numbers. For example, if we were to list the multiples of 4, we would start by multiplying by 1, then by 2, then 3, etc. to get:

\[4, 8, 12, 16, 20, 24, 28, 32, 36, \ldots\]

We cannot list all of the multiples, because we can always multiply by a larger whole number to find another multiple.

What if we wanted to list 5 multiples of 10? We could multiply by 1, then 2, etc. to have the list 10, 20, 30, 40, and 50. Note that we could also list 90 or 150. The request was for 5 multiples of 10, so any 5 will do, but we often just use the first 5.

What are 4 multiples of 12? Multiplying by 1, 2, 3, and 4, respectively, we have 12, 24, 36, and 48. Again, we could have listed other multiples if we preferred.

Now you can practice determining the prime factorization of numbers, and listing multiples. Answers are listed before the exercises.

You Try 6: Determine the prime factorization of each given whole number.

   a. 30
   b. 24
   c. 98

You Try 7: List the **first** 4 multiples of each given whole number.

   a. 11
   b. 25
   c. 19

**VII. Rounding and Estimating**

Now, let’s talk about rounding. When we **round** a number, it means we look for a number that is close to the original number, but is generally easier to work with. We can then use rounded numbers when estimating a sum, difference, product, or quotient. Depending on how accurate we need to be, we can choose a different place value to which we will round.
If we have a number, we can round to whichever place value we need to, but what rules would we need to follow? Given the number 63, what would it become if we round to the nearest ten? Note that the numeral 6 is in the tens place. Rounding to the nearest ten means we want every digit to the right of 6 to be zero. The decision we have to make is regarding the 6. We will either “round down” to the nearest ten, or we will “round up” to the nearest ten. We will look at the digit immediately to the right of the 6 to make our decision. If the number in the ones place is less than 5, we “round down” to 60. If the number in the ones place is 5 or greater, we “round up” to 70. So, if we round 63 to the nearest ten we get 60, because 3 is less than 5.

Now, you can practice rounding numbers. Answers are listed before the exercises.

You Try 8: Round each given number to the indicated place.

a. 873; tens
b. 873; hundreds
c. 53,684; thousands
d. 53,684; ten-thousands

Rounding a number is important because it is often used to estimate a sum or difference. Estimating is used when you want to get an idea of what a sum or difference of numbers is quickly, but don’t need the exact answer. When estimating, round each individual number to the specified place first, then perform the addition or subtraction.

You Try 9: Use rounding to estimate the sum or difference.

\[ \begin{align*}
2533 & \quad \text{Round to the nearest thousand} & 1259 & \\
3187 & \quad \text{Round to the nearest hundred} & 351 & \\
+9676 & & +610 & \\
7568 & \quad \text{Round to the nearest ten} & -234 &
\end{align*} \]

SOLUTIONS TO YOU TRY PROBLEMS:

You Try 1: a. 25,864: \((2 \times 10,000) + (5 \times 1,000) + (8 \times 100) + (6 \times 10) + (4 \times 1)\), twenty-five thousand, eight hundred sixty-four

b. 389,502: \((3 \times 100,000) + (8 \times 10,000) + (9 \times 1,000) + (5 \times 100) + (2 \times 1)\), three hundred eighty-nine thousand, five hundred two

c. 7,100,623: \((7 \times 1,000,000) + (1 \times 100,000) + (6 \times 100) + (2 \times 10) + (3 \times 1)\), seven million, one hundred thousand, six hundred twenty-three

You Try 2: a. 90; b. 801; c. 1402

You Try 3: a. 32; b. 18; c. 107
You Try 4: a. 280; b. 2914; c. 16,605
You Try 5: a. quotient: 8, remainder: 0; b. quotient 31, remainder 3; c. quotient 230, remainder 7; d. quotient 209, remainder 3
You Try 6: a. $2 \times 3 \times 5$; b. $2^3 \times 3$; c. $2 \times 7^2$
You Try 7: a. 11, 22, 33, 44; b. 25, 50, 75, 100; c. 19, 38, 57, 76
You Try 8: a. 870; b. 900; c. 54,000; d. 50,000
You Try 9: a. 16000; b. 2300; c. 7340