Math 273, Written Assignment 1 Name: Grader ID#: ____________

I prefer that you submit your answers on a printed copy of this document, like it's a quiz or exam. However, you may instead rewrite the questions by hand before solving them. Staple sheets together, in order. **Be neat. Always give enough work and clear explanation so that fellow students could follow what you did (from start to finish) just by reading your paper.** Numbers in [ ] give point values for each question.

1. (a) Complete each formula with an expression involving the angle θ between the vectors. (assume $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$):

[1] $\mathbf{a} \cdot \mathbf{b} = |\vec{a}||\vec{b}|\cos\theta$

[1] $|\mathbf{a} \times \mathbf{b}| = |\vec{a}||\vec{b}|\sin\theta$

(b) For $\mathbf{u} = \langle 1, 2, 2 \rangle$ and $\mathbf{v} = \langle 2, -1, 3 \rangle$, find each of the following:

[2] (i) $\mathrm{scal}_{\mathbf{v}}(\mathbf{u}) = |\vec{u}|\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$

$$\frac{\langle 1,2,2\rangle \cdot \langle 2,-1,3\rangle}{|\langle 2,-1,3\rangle|} = \frac{1(2)+2(-1)+2(3)}{\sqrt{\langle 2,-1,3\rangle \cdot \langle 2,-1,3\rangle}} = \frac{2-2+6}{\sqrt{2(2)+(-1)(-1)+3(3)}} = \frac{6}{\sqrt{4+1+9}} = \frac{6}{\sqrt{14}} = \frac{3\sqrt{14}}{7}$$

[2] (ii) $\mathrm{proj}_{\mathbf{v}}(\mathbf{u}) = |\vec{u}|\cos\theta\left(\dfrac{\vec{v}}{|\vec{v}|}\right) = \mathrm{scal}_{\vec{v}}(\vec{u})\left(\dfrac{\vec{v}}{|\vec{v}|}\right) = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\left(\dfrac{\vec{v}}{|\vec{v}|}\right) = \dfrac{3\sqrt{14}}{7}\left(\dfrac{\vec{v}}{|\vec{v}|}\right)$

$$\frac{3\sqrt{14}}{7} \cdot \frac{\langle 2,-1,3\rangle}{\sqrt{14}} = \frac{3}{7}\langle 2,-1,3\rangle = \boxed{\left\langle \frac{6}{7}, -\frac{3}{7}, \frac{9}{7} \right\rangle}$$

[2] (iii) the angle θ between $\mathbf{u}$ and $\mathbf{v}$ (give answer to the nearest 0.1 degrees)

$$|\vec{u}|\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}, \quad \cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}, \quad |\vec{u}| = \sqrt{\langle 1,2,2\rangle\cdot\langle 1,2,2\rangle} = \sqrt{1^2+2^2+2^2} = \sqrt{1+4+4} = \sqrt{9}$$
$$= \sqrt{9} = 3$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} \cdot \frac{1}{|\vec{u}|} = \frac{3\sqrt{14}}{7} \cdot \frac{1}{3} = \frac{\sqrt{14}}{7}, \quad \cos^{-1}\left(\frac{\sqrt{14}}{7}\right) = \theta = 1.006853685 \text{ rad}$$

$$1.006853685 \text{ rad} \cdot \frac{180°}{\pi} = 57.68846676 = 57.7°$$

[2] (iv) the area of the parallelogram with $\mathbf{u}$ and $\mathbf{v}$ as adjacent sides (give answer in the form $\sqrt{\text{integer}}$)

$$|\vec{u}\times\vec{v}| \to \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix}\hat{i} - \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}\hat{j} + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}\hat{k} = (6+2)\hat{i} - (3-4)\hat{j} + (-1-4)\hat{k}$$
$$= 8\hat{i} - (-1)\hat{j} - 5\hat{k} = 8\hat{i} + \hat{j} - 5\hat{k}$$

$$|\langle 8, 1, -5\rangle| = \sqrt{8^2+1^2+(-5)^2} = \sqrt{64+1+25} = \boxed{\sqrt{90} \text{ units}^2}$$

2. Given the lines $L_1 : \mathbf{r}_1(t) = \langle -2+3t,\ 2t,\ 3t\rangle$ and $L_2 : \mathbf{r}_2(s) = \langle -6+s,\ -8+2s,\ -12+3s\rangle$, do the following:

[3] (a) Find the point P where the lines L_1 and L_2 intersect.

$r_1(t) = \langle -2+3t, 2t, 3t\rangle = \langle -6+s, -8+2s, -12+3s\rangle = r_2(s)$

$-2+3t = -6+s$

$2t = -8+2s$

$(3t = -12+3s) = (2t = -8+2s)$

$\boxed{t = -4+s}$

$-2+3(-4+s) = -6+s$

$-2-12+3s = -6+s$

$-14+3s = -6+s$

$2s = 8$

$s = 4$

$-2+3t = -6+4$

$-2+3t = -2$

$3t = 0$

$t = 0$

$\langle -2+3(0), 2(0), 3(0)\rangle = \langle -6+4, -8+2(4), -12+3(4)\rangle$

$\langle -2, 0, 0\rangle = \langle -2, 0, 0\rangle$

$\boxed{P = \langle -2, 0, 0\rangle}$

[3] (b) Find an equation for line L_3 which also passes through point P and is perpendicular to lines L_1 and L_2.

$L_1: \ x = -2 + 3t$
$\quad\ \ y = 0 + 2t$
$\quad\ \ z = 0 + 3t$

$L_2: \ x = -6 + s$
$\quad\ \ y = -8 + 2s$
$\quad\ \ z = -12 + 3s$

$\vec{r} = \vec{r}_0 + t\vec{v}$ directional vector

$$\langle 3, 2, 3\rangle \times \langle 1, 2, 3\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 1 & 2 & 3 \end{vmatrix} =$$

$$\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix}\hat{i} - \begin{vmatrix} 3 & 3 \\ 1 & 3 \end{vmatrix}\hat{j} + \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix}\hat{k} = (6-6)\hat{i} - (9-3)\hat{j} + (6-2)\hat{k}$$

$= 0\hat{i} - 6\hat{j} + 4\hat{k}$

$= \langle 0, -6, 4\rangle$

$\vec{r} = \vec{r}_0 + t\vec{v} \rightarrow \vec{r}_3(t) = \boxed{L_3 = \langle -2, 0, 0\rangle + t\langle 0, -6, 4\rangle = \langle -2, -6t, 4t\rangle}$