Math 202 Fall 2013 Exam 2
November 6, 2013

Covers sections 11.4 - 13.1

- Read all directions carefully.
- You must show all work or justify your reasoning with meaningful evidence to receive full credit, regardless of whether it is explicitly asked for in the question.
- Simplify completely wherever possible.
- Clearly indicate your final answer for each problem and label units appropriately.
- The back page is left blank for you in case you need more space to work through a problem. Please indicate on the exam if you have used the back page for a problem.

1. (4 points each) Find the derivative of the following functions.

11.4 a. \( f(x) = \sqrt{4x^2 + 3x} \)

\[
\frac{d}{dx} \left( (4x^2 + 3x)^{\frac{1}{2}} \right) = \frac{1}{2} (4x^2 + 3x)^{-\frac{1}{2}} \cdot (8x + 3)
\]

11.4 b. \( g(x) = e^{3x^2 - 4x + 7} \)

\[
g'(x) = e^{3x^2 - 4x + 7} \cdot (6x - 4)
\]
2. Let \( h(x) \) be a composite function such that \( h(x) = f[g(x)] \), where \( f \) and \( g \) are defined by \( f(u) = e^u + u^2 \) and \( g(x) = 3x^2 + 1 \).

a. (2 points) Find \( h(x) \).

\[
 h(x) = f(3x^2 + 1) = \left[ e^{3x^2 + 1} + (3x^2 + 1)^2 \right].
\]

b. (2 points) Evaluate \( h(0) \).

\[
 h(0) = e^{3(0)^2 + 1} + (3(0)^2 + 1)^2 = e^1 + 1^2 = e + 1.
\]

c. (2 points) Find \( h'(x) \).

\[
 h'(x) = e^{3x^2 + 1} \cdot (6x) + 2(3x^2 + 1) \cdot 6x.
\]

3. (5 points) Air is being pumped into a spherical balloon at a rate of \(4 \text{ cm}^3/\text{min} \).
Determine the rate at which the radius of the balloon is increasing when the \textbf{diameter} of the balloon is 10 cm. The formula for the volume of a sphere is \( V = \frac{4}{3} \pi r^3 \), where \( r \) is the radius of the sphere.

\[
\frac{dV}{dt} = 4 \text{ cm}^3/\text{min}.
\]

Take \( \frac{dr}{dt} = \frac{4}{3} \pi \cdot 3r^2 \cdot \frac{dr}{dt} \).

\[ \text{At the moment of interest, rate relation becomes} \]

\[
4 = \frac{4}{3} \pi \cdot 3(5)^2 \cdot \frac{dr}{dt}.
\]

So \( \frac{dr}{dt} = \frac{4}{25 \pi} \text{ cm/min} \).

\[ \text{when diameter = 10,} \quad r = 5 \]
4. (4 points each) Given the implicit function, \( x^2 + xy^2 + 5y = 7 \):

a. Differentiate the equation implicitly and then solve for \( y' \).

\[
\frac{d}{dx} \left( x^2 + xy^2 + 5y = 7 \right) \\
2x + x \cdot 2yy' + 1y^2 + 5y' = 0 \\
solve \quad 2xyy' + 5y' = -2x - y^2 \\
y' \left( 2xy + 5 \right) = -2x - y^2 \\
y' = \frac{-2x - y^2}{2xy + 5}
\]

b. Find the slope of all possible tangent line(s) of the function when \( x = 1 \).

When \( x = 1 \), eqn. becomes \( 1 + y^2 + 5y = 7 \), so \( y^2 + 5y - 6 = 0 \),
so \( (y+6)(y-1) = 0 \), so \( y = -6 \) or \( y = 1 \). So two points: (1, -6) and (1, 1).

At (1, -6), slope is \( \frac{-2(1) - (-6)}{2(1) + 5} = \frac{-38}{7} \).

At (1, 1), slope is \( \frac{-2(1) - (1)}{2(1) + 5} = \frac{-3}{7} \).

c. Write an equation for the tangent line(s) of the function when \( x = 1 \).

At (1, -6), tan. line is \( y + 6 = \frac{-38}{7} (x - 1) \).

At (1, 1), tan. line is \( y - 1 = \frac{-3}{7} (x - 1) \).

5. (2 points each) The price-demand equation, with demand \( f \), as a function of price, \( p \), is defined to be \( f(p) = 1000 - 10p \). The elasticity of demand, \( E(p) \), is defined as \( E(p) = \frac{-p f'(p)}{f(p)} \). Write TRUE in front of the correct expressions and FALSE in front of the incorrect expressions.

a. TRUE If the $20 price changes by 10%, then the demand with change by 2.5%.

\[
E(p) = \frac{-p f'(p)}{f(p)} = \frac{1002}{100-10p} . \\
E(20) = \frac{2002}{800} = \frac{1}{4} \approx \frac{\% \text{ change in } x}{\% \text{ change in } p} , \\
\text{so yes.}
\]

b. FALSE When the price is $20, the demand is elastic.

\[
\frac{1}{4} < 1, \text{ inelastic}
\]

c. TRUE When the price is $80, a price increase will decrease revenue.

\[
E(80) = \frac{1}{4}, \text{ elastic}
\]

d. False When the price is $80, a change in price produces a smaller change in demand.

\[
E(80) = \frac{1}{4} \approx \frac{\% \text{ change in demand}}{\% \text{ change in price}} , \\
\text{so } \left( \% \text{ change in demand} \right) \approx 4 \left( \% \text{ change in } p \right)
\]
6. (5 points) Find the absolute maximum and the absolute minimum of the function \( f(x) = x^2 - 6x + 8 \), if they exist.

\[
\text{Not on 2015 spring Exam 2}
\]

Easy though: vertex is at \( \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3 \), opens up, so

\[
\text{NO MAX, MIN is } f(3) = 9 - 18 + 8 = -1.
\]

7. (3 points each) Evaluate the following limits, use L'Hopital's rule where it applies. Be sure to indicate that you are using L'Hopital's rule by writing "L'H" above the "=" sign where you apply the rule.

\[\begin{align*}
\text{a. } \lim_{x \to \infty} \frac{x^3}{\ln(3x)} &\to \infty \quad \text{L'H} \\
&\lim_{x \to \infty} \left( \frac{3x^2}{\frac{1}{3x^2}} \right) \quad \text{algebra} \\
&\lim_{x \to \infty} \left( 3x^3 \right) = \infty.
\end{align*}\]

\[\begin{align*}
\text{b. } \lim_{x \to 0} \frac{e^x - 1}{x^2 - 3x} &\to 0 \quad \text{L'H} \\
&\lim_{x \to 0} \left( \frac{e^x}{2x - 3} \right) \\
&= \left( -\frac{1}{3} \right).
\end{align*}\]
9. (4 points each) Evaluate each indefinite integral.
   a. \( \int (\sqrt{x} - 7e^x) \, dx \)

   Not on Exam 2

   b. \( \int \left( x^{-1} - \frac{8}{x^3} \right) \, dx \)

   Not on Exam 2

10. (5 points) A rectangular fence is constructed adjacent to a farm – house such that the 100 feet of the wall of the house forms part of the fence as shown in the figure. If 400 feet of fencing is required, what is the maximum area that can be enclosed?

Not on Exam 2